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Word Problems Involving Fractions: A Case of Year 5 Pupils in Brunei Darussalam

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ABSTRACT

Word problems involving fractions have always been considered as difficult not only for the students to learn but also for teachers to teach (Jamilah, 2004). This paper reports on how a sample of 37 Year 5 pupils performed in solving word problems involving fractions. Five types of errors as identified by Newman (1977) were committed by the sample students. The five types of errors were reading, comprehension, process, transformation and encoding errors. All those types of errors were profusely exhibited by the sample in the pre-test. A series of intervention lessons were administered to the sample. The approaches used in the intervention lessons were "act it out", "draw a diagram" and make an equation" using concrete, semi-concrete and abstract materials. The post-test result showed that though the improvement was not very highly statistically significant, some improvement in the students' performance were observed. This implies that students need to absorb the concept of basic fractions very well first before they could apply them in solving word problems. The study also suggests that the 3 alternative strategies used worth to be explored further with a bigger sample in future. An implication on the teacher education preparation with regards to the teaching and learning of fractions will also be discussed.

Keywords: Fractions, errors, problem solving

Introduction

The study of fractions is foundational in mathematics, yet it is among the most difficult topics of mathematics for school pupils (Cramer, Behr, Post, & Lesh, 1997; Yusof, 2003). Despite its acknowledged difficulty, children encounter fractions and fraction-related concepts both in real-life and in classroom situations. A firm understanding of fractions undoubtedly helps children make sense of a huge number of other ideas in their daily life. Despite the context in which children engage fractions, it is generally agreed that this topic provides teachers with insight into developments in children's understanding of, and relations among, numbers (Yusof, 2003; Wikkins, 2014). These understandings are built on children's personal experience.

Pupils' errors, if detected at an early stage, should be corrected in order to prevent them from developing further. Errors become evident at any stage and in any topics learnt by pupils. Therefore, it is the responsibility of mathematics educators to attempt to eliminate or at least to minimize this learning problem. An immediate intervention is necessary in order to avoid having those errors become deep seated in the pupils' minds. Traditionally, teachers

use the re-teaching approach to overcome mathematical errors and misconceptions, but Wiggins (2014) stressed the importance of conceptual understanding and suggested that the re-teaching approach has not always resulted in sustained conceptual change and eradication of the error. Currently, alternative programs have been described in the literature that recommends actively using pupils' error patterns/misconceptions as a focal point for intervention and conceptual change (Borasi, 1985). In 1975, Cox concluded from her research that not only did children make systematic errors, but without instructional intervention, they also continued with the error patterns for long periods. Cox emphasizes that teachers must look for patterns in the work they collect from pupils having difficulty with computation as the first step in correcting the error.

Purpose of the Study

The main purpose of the study was to identify the types of errors that Year 5 students made in solving word problems involving fractions. Subsequently, s series of interventions lessons were implemented by introducing alternative strategies in solving word problems involving fractions, in an effort to minimize the errors made by the students and also examine further which of those identified errors were still persistent among the sample students in the post test which was implemented after the intervention lessons.

Research Questions:

The study was guided by the following Research Questions:

1) What types of errors do Year 5 students make in solving word problems involving fractions?
2) Will Year 5 students' performance improve after the intervention lessons?
3) What types of errors are still persistent after the intervention lessons?

Literature Review

Word problems have long constituted a major part of both upper primary and secondary school mathematics both within the instructional programme as well as formal assessment in Brunei. Since 2009 i.e. with the implementation of the new education system called SPN21 (21st Century Education System), Mathematics along with some other subjects like General Studies and Science has been taught in English right from Year 1. Students are required to solve mathematics word problems that are presented in English. Research (Jamilah, 2003) has indicated that both language and semantic structures play an important part in determining students' performance in the solving of mathematical word problems. It was interesting to examine students' various errors in their effort to solve those word problems successfully. For this study, the Newman Errors Analysis (NEA) was used to analyse and discuss those sample students committed in the 12 words problems posed to them both in the pre and post-tests.

The Newman Procedure (1977) is a method that analyzes errors in sentence problems. In the process of problem solving, there are many factors that support the students to arrive at a correct answer. This method supposes that in the process of problem solving there are two kinds of obstacles that hinder students from arriving at correct answers:

1. Problems in linguistic fluency and conceptual understanding that correspond with level of simple reading and understanding meaning of problems, and
2. Problems in mathematical processing that consists of transformation, process skills, and encoding answers.

This classification implies that the students have to interpret the meaning of each question in mathematical context before they can proceed to mathematical processing to obtain

appropriate answer. In summary, Newman Procedure can be described as in Figure 1, which was used as the research framework of the study.

Reading: If students can read the questions	General performance of sample students Comparison among the more able and less able students Comparison of pre and post tests performance
Comprehension: If students can explain	
Transformation: If students can select appropriate mathematical operators and procedures	
Process skills: If students can correctly perform mathematical processing)	
Encoding: (If students can represent answers appropriately)	

Figure 1: Research Framework

Studies such as Jamilah (2003), Raimah (2001), Saman (2000) and See & Jamilah (2000) used NEA (Newman Error Analysis) in identifying the types of errors students were making in solving word problems involving fractions. Their studies found that Year 5 and Year 6 students demonstrated a highest percentage of process and transformation errors. They suggested that those generated results could form a basis for teachers to assist planning for remediation by focussing specifically on those two areas of concern. Those studies further reported that transformation errors hindered students from arriving at the correct solution.

Ismail (2010) stated that when solving a mathematical word problem, cognitive factors contribute to the effectiveness of solving word problems. Teachers are agents in providing activities through the use of rich contexts. Students will fit new experiences with their existing cognitive structure, modifying the structure as needed. It is clear that the difficulties that students have with word problems arise from both the differences in student cognition as well as with the structure and context of the word problems themselves (Lumpkin, 2007).

Various strategies have been introduced in an effort to improve students' performance in solving word problems, not excluding word problems involving fractions. Jitendra (2001) introduced using graphic representation in solving word problems, Jitendra, George, Sood and Price (2010) introduced Schema-Based Instruction (SBI). Both studies have reported the success of the approaches in helping students solving word problems more effectively.

In this study alternative methods in the teaching and learning of problem solving involving fractions were introduced to the sample students in a series of intervention lessons. The strategies were applying the concrete, non-concrete and abstract approaches in the teaching instructions embedded in the "act it out", "draw a diagram" and "make an equation" activities respectively.

Thirty-seven students of Year 5 from 2 mixed-ability classes of one primary government school participated in this study. The students' average age was 11 years old.

Methodology

Instruments

The study was mainly quantitative and supported by qualitative data collected through lessons observation and students interviews. The quantitative data derived from students' responses in the pre and posr-tests.

Instruments:

Three instruments were used to collect the required data, namely; diagnostic pre-test, diagnostic post-test and the students' interview using the Newman protocol. All instruments were piloted-tested first for validity and amendment purposes with a few students who were of the same age range with the main sample but not those participating in the main study. Both the pre and post-tests contained 12 items covering the areas of fractions that the sample students have covered with their actual class teachers.

Data Collection:

After the pre-test was pilot tested for clarity and validity, the 12-item paper and pencil test was implemented to the selected 2 classes of Year 5 pupils on the same day. The sample students were given 30 minutes to complete the test. The 12 items were selected from the pupils' workbook with some minor modification. The pupils' responses were analysed and their errors were categorized according to Newman's 5 types of errors. After the implementation of the intervention lessons, a post-test was carried out to examine if the pupils could solve the word problems better than the pre test and to further examine which of those five types of errors are still persistent. Six students were selected for the interview (Two students of above average, average and below average, based on their performance in the pre- and post-tests), in order to probe further on their attempt in solving the given problems.

Data Analysis

Students' errors were analysed and categorized according to Newman's 5 types of errors. Paired-sample t-test was used to analyze if there was a decrease in means between the pre and post-tests. Means of all error types were also analyzed to examine which of those errors were still persistent after the intervention lessons. Table 1 shows the Mearns, Standard Deviation, t-value, df, Sig value and Effect size between the Pre and Post Tests on Each Type of Errors after Newman (1977).

Table 1

Means, Standard Deviation, t-value, df, Sig value and Effect size between the Pre and Post Tests on Each Type of Errors

	Means	SD	t	df	Sig. value	*D*
Pre R	0.73	1.22	3.25	36	0.00*	0.54
Post R	0.22	0.53				
Post C	4.35	2.24	1.77	36	0.86	0.31
Post C	3.59	2.65				
Pre T	1.30	1.15	-0.34	36	0.73	-0.06
Post T	1.38	1.36				
Pre P	2.14	1.38	0.00	36	1.00	0.00
Post P	2.14	1.80				
Pre E	0.68	0.85	-1.50	36	0.14	-0.27
Post E	0.95	1.15				

Results and Discussion

RQn 1: What types of errors do Year 5 pupils make in solving word problems involving fractions?

Table 1 shows the means of all five types of errors. The result suggests that all the 37 pupils committed all the 5 types of errors. The highest means of errors was on Comprehension (4.35), Process (2.14), Transformation (1.30), Reading (0.73) and Encoding (0.68). This suggests that all students have committed at least one type of error. Some pupils could overcome the reading error, but being able to read the sentence literally did not guarantee them to comprehend the meaning of the questions. They could not further transform, process and encoding the solution successfully. Because of that, they were compelled to manipulate the figures given in the word problems.

RQn 2: Will Year 5 students' performance improve after the intervention lessons?

Means of the pre- and post-tests were compared using the paired-sample t-test and the result shows that there was no statistically significant difference between the pupils' performance (N=37) in the pre-test *(M*= 1.95, *SD* = 1.29) and the post-test (*M* = 2.00, *SD* = 2.10). Further, the effect size value is less than 0.2 (*d* =0.03) suggesting a small effect.

We concluded that on the whole there was a very small and insignificant effect after the 3 intervention lessons as the means difference between the pre and post-tests was not statistically significant.

The possible reason could be the pupils did not have enough exposure to the 3 newly introduced problem-solving methods during the intervention lessons.

The 3 alternative problem-solving strategies were introduced one after the other within a short time gap and it might have created some confusion to the pupils. Some of the more adept pupils were able to adapt gradually although they still demonstrated preferences in using their usual method. This accounted for the small effect after the intervention.

Meanwhile for the less adept pupils, their ability in solving word problems was inhibited mostly by their language difficulties especially in comprehending the wordy sentences.

RQn 3: Which types of errors are still persistent after the intervention lessons?

The means for all types of errors for both the pre- and post-tests were computed using the paired-sample t test. The result shows that though there was a decrease in means for the reading (R) and comprehension errors (C), means for transformation, (T) processing (P) and encoding errors (E) either remained or increased.

Pre R = 0.73 and Post R = 0.22 (Sig Value 0.00*)
Pre C = 4.35 and Post C = 3.59 (Sig Value 0.86)
Pre T = 1.30 and Post T = 1.38 (Sig Value 0.73)
Pre P = 2.14 and Post P = 2.14 (Sig Value 1.00)
Pre E = 0.68 and Post E = 1.15 (Sig Value 0.14)

Therefore, it shows that though there was a significant difference in the reading error and also a slight improvement in the comprehension error, the errors in transformation, processing and encoding errors were still persistent.

This could be due the pupils' English language difficulty as Dockrell and McShane (1992) termed reading difficulties as a failure to recognize written words, which in turn adversely affects reading comprehension and draws on the individual acquired knowledge. If pupils can be helped to improve their reading and comprehension skills, they would be able to proceed to the next stage of executing word problems.

As for the transformation, processing and encoding errors, the possible reason for the no improvement could be the word problem sentence structure made the pupils not able to transform them into the right choice of mathematical operation, thus they failed to process and eventually encoding the correct solution.

Limitation

This study had some limitations. Firstly was the time constraint in data collection. The data had to be collected during the school term when the students were doing fractions according to their term scheme of work, hence within two weeks in their second term. Due to time factor, the study couldn't be extended to a larger sample, thus limited to one school only with 37 Year 5 students. The intervention lessons were not that effective as each of the three alternative strategies was only done once in each lesson. Given more time, a longitudinal study with more series of intervention lessons could have been done.

Conclusion

There are several points teachers need to consider in teaching word problems involving fractions. First, reading for comprehension is a vital aspect as it is the starting point in the process of executing word problems successfully. Secondly, teachers must be aware on the pupils' reading ability and pupils' English language proficiency. Thirdly, is also important to note the different types of errors committed by the pupils as not every pupil experienced the same types of errors in solving the word problems. Finally, in addressing the pupils' errors, teachers need to be aware of those five types of errors and to provide immediate help and remediation accordingly.

To conclude, though the findings of this study suggest that the sample pupils were generally weak in solving word problems involving fractions, the result cannot be generalized to the whole population of Year 5 pupils in Brunei.

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