8 ICLLCE 2017-009 Low Chee Soon

Learning Mathematical Flexibility in a Constructivist 5E Model

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Abstract

Flexibility is a key dimension of creativity. Classroom discourse, which promotes mathematical flexibility, has been highly valued. Past studies have established evidences that learning mathematical flexibility, e.g. by attempting and discussing alternative solutions to multiple-solution tasks, leads to enhanced mathematical learning and performance. However, few studies have addressed how such learning should take place systematically for an optimal result. This preliminary study explored the learning of mathematical flexibility in an adapted constructivist 5E learning model. Conceptual variability in nine purposively sampled A-Level participants' solution strategies for solving three multiple-solution tasks pertinent to Quadratics were qualitatively examined and compared before and after the 5E intervention. The findings generally revealed the participants' enhanced flexibility and accuracy in producing conceptually-varied solutions to the quadratic tasks upon intervention. The constructivist 5E learning of mathematical flexibility provides a guide to classroom discourse as to how the learning of mathematical flexibility may systematically take place.

Keywords: mathematical flexibility, constructivist 5E learning model, conceptually-varied solutions to multiple-solution tasks

Introduction

When flexibility comes into play, many mathematical tasks are self-evidently multiple-solution tasks which can either lead to a wide range of possible answers or be solved in two or more ways. For instance, 'What two integers have a sum of 10?' at the primary level, 'What are the dimensions of a rhombus whose area is 10 cm2?' at the lower secondary level, and 'What is the shortest distance from the point (2, 5) to the line y = 3x + 8?' at the high school level, are all multiple-solution tasks. The teaching and learning of mathematical flexibility is critical and highly valued. National Council of Teachers of Mathematics (NCTM, 1991, 2000) asserts that students should be engaged in mathematical discourse about problem solving which includes discussing different solutions and solution strategies for a given problem. Similarly, the National Research Council (1989) emphasizes that mathematical learning should entail motivation for moving beyond just mathematical rules to also focus on seeking solutions (i.e. not just a solution by memorizing procedures), exploring patterns (i.e. not just memorizing formulas) and formulating conjectures (i.e. not just doing exercises). Numerous studies have established the evidence of potential benefits students may gain from exposure to alternative solutions, flexible use of strategies, and deliberate exploration and comparison of possible solutions (Elia, Heuvel-Panhuizen, & Kolovou, 2009; Greer, 2009; Heinze, Star, & Verschaffel, 2009; Newton, Star, & Lynch, 2010Rittle-Johnson & Star, 2007, 2009; Star & Rittle-Johnson, 2007). In addition, students having knowledge of multiple solutions have been found to make instructional interventions more effective (Alibali, 1999; Siegler, 1995), and are better able to deal with both near- and far-transfer problems (Hiebert & Carpenter, 1992). However, few studies have addressed the pedagogical aspects of teaching mathematical flexibility. Consequently, teachers lack an informed decision as to how students could gain flexible knowledge more systematically.

Objective

This study explored the pedagogical impact of learning mathematical flexibility in an adapted constructivist 5E learning model as intervention. Specifically, the study aimed to qualitatively explore the change in nine purposively-sampled A-Level participants' ability to employ alternative solution strategies for three quadratic tasks as well as the change in the level of precision of their solutions upon the intervention.

Research Questions

In particular, this study aimed to answer the following questions:

1) How will the A-Level participants' ability to produce multiple solutions to quadratic tasks change upon the intervention?
2) How will the accuracy of their solutions change upon the intervention?

Theory

Mathematical flexibility. Flexibility is a key attribute of creativity. It primarily refers to switching smoothly between different strategies (Guilford, 1959; Selter, 2009; Stein, 1974; Torrance, 1969). Verschaffel, Luwel, Torbeyns and Dooren (2009) referred to flexibility as use of multiple strategies. Star and Rittle-Johnson (2007) as well as Star and Seifert (2006), however, defined flexibility in solving linear equations as knowledge of (a) multiple strategies and (b) the relative efficiency of these strategies, perceiving efficiency as an intrinsic attribute of flexibility.

In this study, we view mathematical flexibility as the ability to employ, in particular, conceptually-varied solutions for multiple-solution tasks (Low, 2015). Conceptuallyvaried mathematical solutions do not differ only in procedures (e.g. simply different orders in solution steps) but also the use of different concepts in arriving at the solutions.

Constructivist 5E Learning Model. The constructivist 5E learning cycle is a research-oriented, phase-based instructional model which lends systematic strategies to instructional implementation. The 5E learning cycle consists of five phases, namely engagement, exploration, explanation, elaboration, and evaluation. Bybee, Taylor, Gardner, Scotter, Powell, Westbrook and Landes (2006) elucidate the essential nature and the instructional objectives of the five phases. Further details follow.

Engagement phase. In general, this phase is aimed at arousing the students' curiosity so that they could be motivated and gain a sustaining interest in the instructional task in hand, and are in a mode ready for further exploration. In addition, it is also to expose students' misconceptions, if any, so that cognitive disequilibrium could be made sufficiently explicit to be dealt with in the subsequent phases.

Exploration phase. Being engaged, the students then begin to explore concepts, ideas, relationships, patterns, etc. and gain hands-on experience in dealing with the task in hand. In the exploration phase, the instructor only serves as a facilitator to orientate students' exploration without providing any explicit solutions. However, students ought to be given sufficient encouragement and freedom to explore own ideas and thinking.

Explanation phase. This phase sets a platform for the students to share their observations and beliefs, followed by the instructor's clarifications when needs arise. It then serves to help students in ordering their exploratory experiences and mental structures. Any relevant concepts, processes, or skills have to be presented concisely but clearly before the students move on to the subsequent phases.

Elaboration phase. In this phase, attempts will be made to facilitate transfer of concepts to closely related but new situations. Students can be engaged in group discussion, giving opportunity for individual students to extend the ideas learned earlier and to elaborate on the conception of the task and other strategies which could possibly contribute to the task completion. Students ultimately gain the opportunity, via group discussions and cooperative learning, to express their understanding of the subject under study, exchange ideas and learn from peers of similar levels of understanding. More critically, this elaboration phase allows for transfer of identical explanations and generalization of concepts, process, and skills.

Evaluation phase. Evaluation could occur across all phases. It provides the opportunity for students to evaluate their understanding. Be it a formative or summative evaluation, the students should receive feedback either from the more capable peers or the instructor so as to ascertain the achievement of the required educational outcomes.

The 5E learning model is fundamentally based on the constructivist premise that interaction exists between a learner’s prior knowledge and the target learning contents in the formation of new knowledge, and that the efficacy of knowledge construction also relies on social interaction among peers and the instructor. As such, it is critical to positively engage learners so that they could sustain their interest to explore ideas and have the opportunities to share and express their understandings, both individually and in groups. In other words, the constructivist perspective focuses on practical exploratory knowledge construction, which is believed to be critical for conceptual development, rather than passive theoretical knowledge transfer. Equally emphasized is the need to evaluate the learning process throughout the entire learning cycle. In short, constructivist inquiry learning undergirding the 5E model underlies not only the declarative content knowledge, but also the pedagogical content knowledge, which would assist learners in constructing meaningful concepts and knowledge. Experimental results generally demonstrate strong and statistically significant gains in student achievement, across varying student abilities (Pinkerton & Stennet, 2007).

Theoretical framework of the study: Figure 1 shows the theoretical framework which embeds the learning of mathematical flexibility in an adapted constructivist 5E learning model. While most studies have employed the 5E model sequentially particularly in scientific inquiry, other educators and researchers, such as Eisenkraft (2003) and TUNA and KAÇAR (2013), have suggested that the 5E learning cycle may not be necessarily linear. In particular, they assert that formative evaluation should not be exclusive to a particular phase of the learning cycle, but must take place in every phase during all interactions with students.

In this study, we contend that, unlike scientific inquiry, learners could be nonlinearly led into any phase of the 5E cycle flexibly and conditionally, taking into such considerations as the nature and complexity of mathematical contents and explorations, the participants’ prior knowledge, response and needs, as well as the availability of time in a lesson. For instance, when misconception of an idea surfaces during an explanation phase, learners could be led to further explore (i.e. into exploration phase) the idea, re-engaged (i.e. into engagement phase) by scaffolding the discussion, at the same time, required to critically evaluate (i.e. into evaluation phase) by comparing and contrasting the faulty conception with the emerging new ideas. This explains the flexible nonlinear pplication of the 5E learning model in this study as illustrated in Figure 1.

Table 1 illustrates the leading questions for orientating the learning of mathematical flexibility via the adapted 5E learning model, which serves as a pedagogical tool for systematically yet flexibly orientating the instructional phases to facilitate students in dealing with multiple-solution tasks.

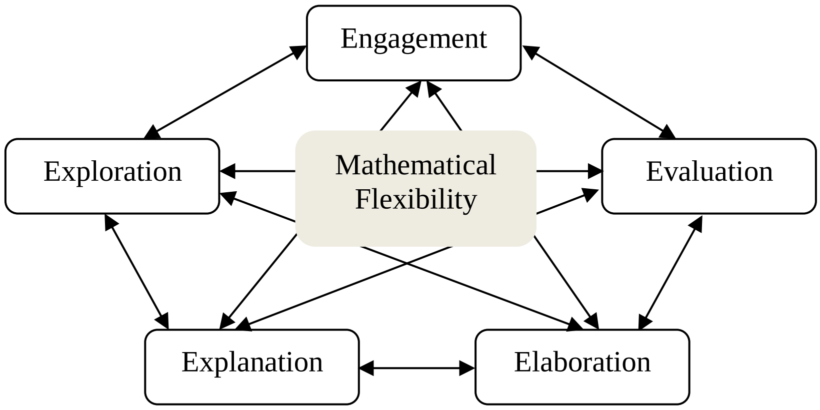

Figure 1. Flexible nonlinear application of the 5E learning model

Table 1
Leading questions for mobilizing the 5E learning of mathematical flexibility

The 5 E's	Leading Questions
Engage To engage students, elicit their prior knowledge, arouse their interest and curiosity, set the mode for further exploration	What do you think is the solution to this task? Do you believe it can be solved in more than one way? Can multiple-solution strategies be employed?
Explore To explore possible solutions to the task in hand, determine and implement a strategy to see if a solution works, explore the relations of the solution to the pertinent mathematical concepts/properties/operations	What mathematical properties/concepts/operations have you employed? What are other solution strategies that could be applicable? Do the employed strategies work? How would you relate your solutions to the pertinent mathematical concepts?
Explain To explain the solutions and the strategies employed, and their appropriateness	How/Why do your strategies work or not work? Why is a particular strategy adopted or deemed appropriate? How did you think of using this strategy? Why do you like or dislike the solution strategies?
Elaborate To extend to other possibilities or alternative solutions, attempt to interpret the problem features from different perspectives or in a new light	Could there be any other solution strategies that are equally applicable? Can other mathematical concepts/properties be possibly relevant and applicable? Can you apply your strategies to other dissimilar but conceptually-similar tasks? Can your solutions be possibly 'trimmed' so that they look more efficient and elegant?

Evaluate	
To evaluate the attempted solution strategies, their implications, advantages and disadvantages	What are the advantages or disadvantages of the solutions you have attempted? How could each solution be enhanced? Do you think all the solution strategies employed have the same quality, i.e. efficiency, elegance, intelligibility? What is your most preferred solution? Why?

Literature Review

Mathematical Flexibility

The importance of promoting mathematical flexibility by emphasizing varieties of solutions, thus allowing freedom of choice in student learning, has received considerable attention even since a few decades ago (Freeman, Butcher, & Christie, 1971; Fremont, 1969; Rogers, 1983). Inculcating the predisposition toward possible strategies with profound reasoning rather than just emphasizing correct strategies in the learning culture has been a primary concern in mathematics education (Baron, 1988; Garnham & Oakhill, 1994; Lithner, 2003, 2008; Stacey & Vincent, 2009; Stein, Grover, & Henningsen, 1996). "By encouraging diverse strategic solutions and requiring students to explain these to others, children will realize that there can be more than one way to work things out and that mathematics is about methods as much as it is about right answers." (Cowan, 2003, p. 44).

Peterson (1988) argued that "most higher-level mathematics responses have more than one right strategy that can be used to obtain the answer" (p. 11). As such, instructional focus should not aim at merely getting the right answers, but various strategies for effective problem solving. Silver, Ghousseini, Gosen, Charalambous and Strawhun (2005) claimed that comparing, reflecting on, and discussing multiple solution methods would improve student learning. Pupils' low competence in using different representations of functions in problem solving had been attributed to the lack of flexibility in approaching functions (Elia, Panaoura, Eracleous, & Gagatsis, 2007).

Despite the copious research evidence pointing to the potential benefits of mathematical flexibility in mathematics learning, few studies have ventured into the pedagogic aspects of learning mathematical flexibility. What learning process would lend itself well to the learning of mathematical flexibility? While mathematical flexibility, e.g. the ability to produce alternative solutions to a task, may require substantial engagement to explore and reason about possible solutions which in turn provide learning opportunity by comparing and evaluating different solutions (Rittle-Johnson & Star, 2007, 2009), the constructivist 5E learning model appears to be a promising framework for learning mathematical flexibility.

Constructivist 5E Learning Model

The 5E learning model has been ubiquitously employed since its inception in the mid-1980s and proven to be effective particularly for the learning of Sciences, which lend themselves well to inquiry-based experimental studies. The profound influence of the 5E learning model in the educational arena has been prominent (Bybee et al., 2006).

The increasing focus on the use of constructivist, inquiry-based (such as 5E) learning models in teacher's professional development programs convincingly speaks of its prominent influence (Browning, 2013; Hanuscin & Lee, 2008; Nuangchalerm, 2012; Pasley, Kannapel, & Fulp, 2010; Yalcin & Bayrakceken, 2010). Research findings and reports on the effectiveness of the 5E learning model abound, especially in Science learning involving scientific inquiry (Bybee et al., 2006).

Türk and Çalık (2008) shared the use of different conceptual change methods (i.e. conceptual change text, analogy and worksheet) embedded in a 5E learning model which was claimed to be effective for teaching the nuances of endothermic-exothermic reactions.

Appamaraka, Suksringarm and Singseewo (2009) studied the effects of the 5E learning cycle with the metacognitive moves (characterized by intelligibility, plausibility and wide-applicability) and the teacher's handbook approach on learning achievement, integrated science process skills and critical thinking of ninth-grade students. The study involved eighty-two students, half of which were randomly assigned to an experimental group with the metacognitive moves embedded in the 5E learning cycle, while the other half to a control group associated with the teacher's handbook approach. Findings revealed that the experimental group generally showed better academic achievement, higher integrated science process and critical thinking skills. The effectiveness of the 5E learning model has been similarly confirmed in an investigation by Haribhai and Dhirenkumar (2012) who compared two instructional methods, namely the constructivist 5E instructional model and the traditional lecture method, involving students from both urban and rural areas as moderating variables. Findings indicated that the use of constructivist 5E instructional model positively led to better achievement and retention for students from both urban and rural areas.

Although little evidence is available to reveal the application of 5E model in domains other than the hard sciences, there is an increasing acceptance of it for the learning of such domains as mathematics and environmental education (Appamaraka et al., 2009; Birisci & Metin, 2010; Bybee et al., 2006; Nayak, 2013; TUNA & KAÇAR, 2013). TUNA and KAÇAR (2013) investigated the effect of 5E learning model in teaching Trigonometry on students' academic achievement and knowledge sustainability. Forty-nine tenth-grade students were involved, from which 25 students were randomly assigned to an experimental group and the remaining to a control group, based on their mathematics scores for the last (autumn) semester examination in 2009-2010 academic year as well as the scores in the pre-test prior to instructional intervention. Similar to the findings from other studies, results indicated that the 5E learning model contributed positively to both student achievement and permanence of knowledge.

The opportunities from experiential and cooperative learning within a 5E framework could be very much beneficial. Robertson, Meyer and Wilkerson (2012) creatively designed an outdoor skateboarding activity based on the 5E learning model. In the learning process, students were engaged to practically explore both the mechanical and mathematical concepts (such as Forces, Motion, Newton's Laws of Motion) involved in skateboarding via observation, data collection, group discussion, the opportunity to explain individual's exploration and understandings, evidence-based inference in arriving meaningful conclusions, as well as own and instructor-led evaluations. The activity was found to be really exciting and engaging to learners and was deemed to be successful in ensuring deep conceptual understanding in bridging Algebra and Geometry in the real-life context of Mechanics study.

Methodology

Research Design

This study adopted a qualitative research design whereby the number and accuracy of nine A-Level participants' solution strategies for three pairs of equivalent Quadratic tasks were compared throughout the 5E intervention over three separate sessions.

Participants

Nine A-Level students (5 males 4 females) participated in the study on purposive sampling and voluntary basis. They were all 18 to 19 years old and had just completed their secondary school studies. They were separated into groups of three based on their levels of flexibility assessed in the study by Low (2015). These participants were coded as L1, L2 and L3 (low flexibility group); M1, M2 and M3 (medium flexibility group); H1, H2 and H3 (high flexibility group). Such groupings ensured similar levels of understanding among group members.

Measure

The quadratic multiple-solution tasks employed were:

Before intervention

1a) Express the function $2x^2 - x - 6$ in the form $a(x + b)^2 + c$, where a, b and c are constants.

2a) Find the stationary value of the function $2x^2 - x - 6$, and its corresponding value of x. State if the stationary value of the function is a minimum or a maximum.

3a) Solve the inequality: $(x - 3)^2 - 16 > 0$.

After intervention

1b) Express the function $8 - 2x - 3x^2$ in the form $a(x + b)^2 + c$, where a, b and c are constants.

2b) Find the coordinates of the stationary point on the curve of function $8 - 2x - 3x^2$. State if the stationary point is a minimum or maximum point.

3b) Solve the inequality: $1 - (2x - 3)^2 \geq 0$.

The Quadratics domain was employed in view of its moderate level of complexity and the prior knowledge of the participants. Quadratic functions are neither too simplistic as linear functions nor overly complex as other transcendental functions. The participants had learned about Quadratics from their secondary studies. However, the extent to which they could deal with Quadratics flexibly was initially unknown.

Procedure

Setting: The nine participants were seated in the same row in groups of three. Three in a group facilitated discussion and exchange of ideas among the group members. Two video-recorders were in place, one facing the front board and the other facing the participants. Each group was given a video-recorder so that the group conversations could be captured for further analysis. However, this paper presents only the findings on the written multiple solutions of the participants.

Process: The participants underwent three sessions of learning mathematical flexibility with the adapted 5E learning model (i.e. Session 1: Tasks 1a and 1b, Session 2: Tasks 2a and 2b, and Session 3: Tasks 3a and 3b), on two consecutive days. At the beginning of the study, the participants were briefed on the adapted 5E learning model and assured of confidentiality. It was explained that a multiple-solution task could be solved in two or more ways with different concepts. In each session, the participants were guided by a series of activities with proposed time limits (Table 2), which however were not meant for a slavish adherence. The participants were encouraged to freely explore possible solutions to the given multiple-solution task in hand without the need to strictly adhere to the proposed time limits, which merely served as a guide to avoid any unnecessary delay or prolongation of an activity. However, no time limit was explicitly proposed for whole-group discussions of multiple solutions so that the participants would enjoy the sharing and learning of ideas among each other without time pressure.

Table 2
Instructions for triggering the 5E learning activities with proposed time limits

Instruction	Time allowed (minutes)
1. Attempt the given task in your most familiar and preferred way, and label it ‘Solution 1’.	1
2. Explore if the task can be solved in other way(s), i.e. if other solution strategy/strategies can be employed to solve the task. Label them ‘Solution 2’, ‘Solution 3’, etc.	3
3. Explain your solution strategies to your group members.	2 – 3
4. Evaluate the various solution strategies and state the reason(s) for your preferred solution(s).	0.5
5. Discuss and explain the solution strategies with the whole class.	–

Table 2 shows the typical instructions which triggered the various activities pertinent to the 5E learning model and the proposed time limits for the respective activities. In Table 2, Instructions 1, 2, and 4 were meant for individual activities, Instruction 3 for separate group activities, while Instruction 5 for whole-group activity, which was led by the first author. During any one activity, the prompt questions as shown in Table 1 were employed. The participants were not allowed to add any other solutions to their scripts during both within-group and whole-group discussions. They were however permitted to record any emerging ideas and solutions on their own paper for the sake of learning and future reference. The nonlinear nature of the adapted 5E learning model (Figure 1) was particularly relevant during the whole-group discussions. The times spent on the three 5E sessions ranged approximately from 37 to 71 minutes.

Sequence of tasks: Figure 2 illustrates the sequence of tasks administered and activities (see Table 2) which took place over the three sessions of the 5E learning process. Unlike Tasks 1a, 2a and 3a, Tasks 1b, 2b and 3b were attempted without activity 3 (i.e. individual group discussion) so that the change in solution strategies could be assessed for each participant without group influence.

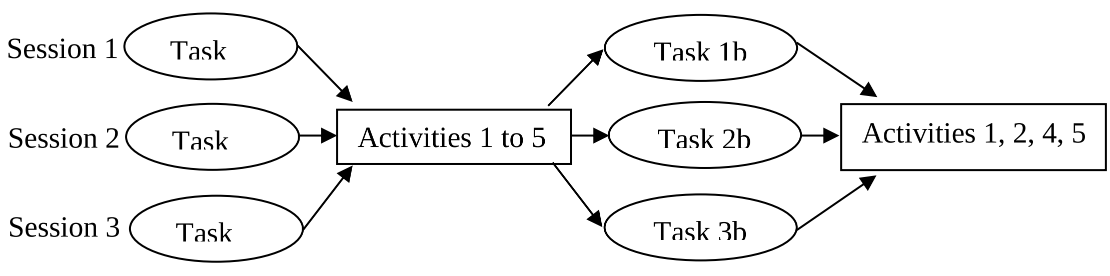

Figure 2 Sequence of tasks and activities over three 5E sessions

Coding and Data Analysis

The participants' solution strategies for each task, including those having emerged during the whole-group discussions, were categorized based upon their conceptual variations labeled 1, 2, 3, etc., as shown in Table 3. Similarly, the levels of accuracy for the participants' solution strategies were classified as low (L), medium (M) and high (H).

Table 3
Emergent solution strategies for the multiple-solution tasks

Task	Solution Strategy
1 Completing-The-Square	1. Completing the square by introducing a constant. 2. Completing the square by reverse expansion. 3. Completing the square by comparing coefficients.
2 Stationary functional value	1. Deriving stationary functional value by completing the square. 2. Deriving stationary functional value by differentiation and zero gradient. 3. Deriving stationary functional value by property of symmetry. 4. Deriving stationary functional value by use of formula.
3 Solving quadratic inequality	1. Expansion →Factorization → Graphical analysis 2. Expansion → Factorization → Factor sign analysis 3. Factorization by difference of two squares → Graphical analysis 4. Factorization by difference of two squares → Factor sign analysis 5. Concept of real values and their magnitudes

The participants' solution strategies were independently coded using the descriptors in Table 3 and rated as L, M, or H, by the first and the third authors. Those solution strategies, which were conceptually incorrect or strategy-unidentifiable due to incompleteness were ignored. The Kappa Measure of Agreement values for (i) the types and (ii) the levels of accuracy of solution strategies before (n = 72) and after (n = 108) intervention were (i) 0.975 and 0.962, and (ii) 0.870 and 0.795, respectively. These levels of inter-rater agreement are generally very good.

Findings

The change in A-Level participants' ability to produce multiple solutions

This study generally revealed the participants' enhanced mathematical flexibility in producing multiple solutions to the administered Quadratic tasks, at both individual and group levels throughout the 5E intervention (Table 4). The group numbers of multiple solutions produced had increased from 7 to 21 (200%) on Task 1, 16 to 19 (19%) on Task 2 and 14 to 25 (79%) on Task 3 upon intervention. Individual participants' numbers of solutions had increased across tasks even before intervention, namely 0 or 1 solution on Task 1, 1 or 2 solutions on Task 2, and 1 to 3 solutions on Task 3. Upon intervention, they produced 2 to 3 solutions on Task 1 and 1 to 4 solutions on both Tasks 2 and 3.

Further analysis revealed other noteworthy aspects. While most participants had extended their repertoire of solution strategies, L1 in Tasks 1 and 3 and M2 in Task 2 had shown the abandonment of prior strategies in adoption of new strategies. In addition, the participants of initial higher flexibility seemed to gain more flexibility upon intervention.

The change in accuracy of solutions upon intervention

The participants were better able to produce accurate solutions upon intervention (Table 4). For instance, five participants, who produced invalid solutions or solutions of low accuracy before intervention, were able to produce at least one solution of high accuracy upon intervention. Similarly, the solutions of four participants changed from medium to high accuracy upon intervention. Most other solutions were of high accuracy throughout the intervention. L1's switch of strategy on Task 3 was the only exception.

Table 4

Participants' solution strategies for multiple-solution tasks

Task	Participant	Before Intervention						After Intervention					
		Solution Strategy						Solution Strategy					
		1	2	3	4	5	Total	1	2	3	4	5	Total
1 Completing-the-square	L1	L					1		H	H			2
	L2						0		H	M			2
	L3						0		L	H			2
	M1	L					1	M		H			2
	M2	L					1	H	H				2
	M3	H					1	H	H	H			3
	H1	H					1	H		H			2
	H2	M					1	H	H	H			3
	H3	H					1	H	H	H			3
							7						21
2 Stationary Functional Value	L1	H	H				2	H		H			2
	L2	M	M				2	H					1
	L3	H	H				2	H					1
	M1		H				1	H	H				2
	M2		M				1	H					1
	M3	H	H				2	H	H				2
	H1	H	H				2	H	H				2
	H2	H	H				2	H	H	H	H		4
	H3	H	H				2	H	H	H	H		4
							16						19
3 Solving Quadratic Inequality	L1	H					1					L	1
	L2	H					1	H				H	2
	L3					H	1	H				H	2
	M1	H					1	H		H		H	3
	M2	H				H	2	H				H	2
	M3	H					1	H		H		H	3
	H1	H	H				2	H	H	H		H	4
	H2	H		H		M	3	H		H	H	H	4
	H3	H		H			2	H	H	H		H	4
							14						25

Level of accuracy: H(high), M(medium), L(low)

Discussion

The findings from this study suggest that mathematical flexibility and accuracy can be nurtured via a nonlinear 5E learning model. While the effectiveness of multiple-solution approach to learning could be highly context-dependent (GroBe & Renkl, 2006), the adapted 5E learning model seemed to have provided a context supportive of the nurturance of mathematical flexibility. In particular, the 5E learning model enables inquiry, investigation, and sound reasoning which are critical learning experiences (Pasley, et al., 2010). The nonlinear nature of the adapted 5E learning model has enabled both the participants and the teacher to freely express and explain ideas which naturally allow for formative assessment and remedial explanations whenever deemed appropriate and necessary without being rigidly adhered to *planned* instructional contents and flows. Such opportunity for both students and instructor to constructively express, explain, evaluate, and clarify ideas is an important characteristic in meaningful classroom discourse (Coultas, 2007; Pressley, Wood, Woloshyn, Martin, King, & Menke, 1992). Both conceptual and procedural knowledge could become plain, comprehensible, and clear, especially during an explanation phase (Bybee et al., 2006).

Weaving through the 5E phases in exploring multiple solutions to a task is not without challenges though. The process of learning multiple solutions to a task was found to be extremely time-consuming and demanding particularly with increased complexity and variability of solutions (i.e. Task 3 in comparison with Tasks 1 and 2). While a 5E learning environment provides the opportunity for the participants to exchange ideas and share their respective solution strategies, the extent to which the participants would learn from one another may vary depending on what they actually valued: Sharing their own thinking to others or learning from what others have to share (Young-Loveridge, Taylor, & Hawera, 2005). Furthermore, the characteristics of multiple-solution tasks and their intended cognitive demand may vary from curricular planning to curricular implementation in the classroom (Stein et al., 1996). Thus, ensuring the optimal effects of instructions on mathematical flexibility could be a highly complex and multi-faceted process which requires more informed decisions from substantial research.

Limitations

The small number of participants has weakened the extent of external validity of this study. In addition, it is inconclusive as to whether tasks of different nature and complexity would produce similar results with the 5E learning model.

Recommendation

While this preliminary study has suggested the efficacy of learning mathematical flexibility in an adapted 5E learning model, further studies could be extended to reach higher numbers of learners of different levels of education, include mathematical domains of different nature and complexity, and even explore the existence of possible moderating variables, such as teacher’s flexible knowledge.

Conclusion

Mathematical tasks are flexibly solvable in two or more ways which may relate to various concepts. This study revealed that students guided by an adapted 5E learning model may reach higher levels of mathematical flexibility and precision. The 5E learning opportunity in social interaction, exploratory inclination and critical evaluation are likely to have contributed to the enhanced mathematical flexibility. However, while few studies have explored contextual influence on the learning of mathematical flexibility, more research is required to ascertain the key contextual attributes for enhanced mathematical flexibility. This study hopefully will instill further interest in educators to explore contextual influence on the learning of mathematical flexibility at more refined levels.

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